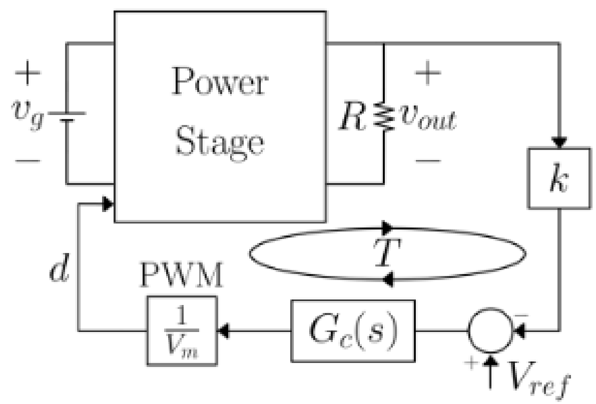
D4 Regulator Classical Control Design

Regulator configuration:



Block diagram of the voltage regulator system with a feedback loop $T(s)$ consisting of the D4 converter power stage characterized by its control to output transfer function $G_{vd} = \hat{v}_{out}/\hat{d}$, resistive divider gain k, compensator transfer function $G_c(s)$, and PWM gain $F_M = 1/V_M$.

Use the SSA (state space averaging) model to determine the control to output transfer function.

State Space Averaging (SSA) Model:

The large signal model is given by:

$$\dot{x} = Ax + Bu \qquad y = Cx + Eu$$

where

$$A = DA_1 + D'A_2 \qquad B = DB_1 + D'B_2$$

$$C = DC_1 + D'C_2 \qquad E = DE_1 + D'E_2$$

$$x = \begin{bmatrix} i_1 \\ i_2 \\ v_1 \\ v_2 \end{bmatrix} \qquad u = v_g$$

X is the steady state vector and U is V_g, the DC value of the input voltage.

$$X = -A^{-1}BU$$

For the transient simulations we will use the small-signal SSA model:

$$\dot{\hat{x}} = A\hat{x} + B\hat{u} + B_d\hat{d}$$

$$\hat{y} = C\hat{x} + E\hat{u} + E_d\hat{d}$$

where matrices A, B, C and E where previously defined and where

$$B_d = (A_1 - A_2)X + (B_1 - B_2)U$$

$$E_d = (C_1 - C_2)X + (E_1 - E_2)U$$

➔

$$\boldsymbol{G_{vd}(s) = C(sI - A)^{-1}B_d + E_d}$$

Using this model results in the small-signal control to output transfer function, $G_{vd}(s)$

Factoring the control to output transfer function:

$$G_{vd}(s) = \frac{\hat{v}}{\hat{d}}(s) = \frac{V_g\left[L_1C_1 s^2 + D(1-D)\frac{L_1}{R}s + 1\right]}{C_1C_2L_1L_2 s^4 + L_1C_1\frac{L_2}{R}s^3 + \left[(1-D)^2C_2L_1 + L_2C_2 + L_1C_1\right]s^2 + \left[(1-D)^2\frac{L_1}{R} + \frac{L_2}{R}\right]s + 1}$$

GIVEN $\quad k\ \left(=\frac{1}{5}\right)\ ;\quad \frac{1}{V_m} = \frac{1}{0.6} = F_m$

APPROXIMATE FACTORIZATION:

$$DEN \approx \left(L_1C_1s^2 + \frac{(1-D)^2L_1}{R}s + 1\right)\left(L_2C_2s^2 + \frac{L_2}{R}s + 1\right)$$

$$= L_1C_1L_2C_2s^4 + \left[\frac{L_1C_1L_2}{R} + \frac{L_2C_2(1-D)^2L_1}{R}\right]s^3 + \left[L_1C_1 + L_2C_2 + \frac{(1-D)^2}{R^2}L_1L_2\right]s^2 + \left[\frac{(1-D)^2L_1}{R} + \frac{L_2}{R}\right]s + 1$$

TO BE ACCURATE

REQUIRE:

1) $\frac{L_2C_2(1-D)^2L_1}{R} \ll \frac{L_1C_1L_2}{R} \Rightarrow (1-D)^2C_2 \ll C_1$

2) $\frac{(1-D)^2}{R^2}L_1L_2 \ll L_1C_1 + L_2C_2 \Rightarrow$ if $L_2 \gg L_1 \Rightarrow \frac{(1-D)^2}{R^2}L_1L_2 \ll L_2C_1 \Rightarrow \frac{(1-D)^2}{R^2}L_1 \ll L_2$ ✓

3) $(1-D)^2C_2L_1 \ll L_2C_2 + L_1C_1 \Rightarrow$ if $L_2 \gg L_1 \Rightarrow (1-D)^2C_2L_1 \ll L_2C_2$

$\Rightarrow (1-D)^2L_1 \ll L_2$ ✓

Adding losses:

$$G_{v_d}(s) = \frac{V_g\left(L_1C_1 s^2 + D(1-D)\frac{L_1}{R}s + 1\right)}{\left(L_1C_1 s^2 + \frac{(1-D)^2L_1}{R}s + 1\right)\left(L_2C_2 s^2 + \frac{L_2}{R}s + 1\right)}$$

$$sL_1 \to sL_1 + r_1 \quad ; \quad sL_2 \to sL_2 + r_2$$

$$sC_1 \to \frac{sC_1}{1+sr_{c_1}C_1} \quad ; \quad sC_2 \to \frac{sC_2}{1+sr_{c2}C_2}$$

$$\Rightarrow \; = \frac{V_g\left[(sL_1+r_1)\left(\frac{sC_1}{1+sr_{c_1}C_1}\right) + D(1-D)\frac{(sL_1+r_1)}{R} + 1\right]}{\left[(sL_1+r_1)\left(\frac{sC_1}{1+sr_{c_1}C_1}\right) + \frac{(1-D)^2(sL_1+r_1)}{R} + 1\right]\left[(sL_2+r_2)\left(\frac{sC_2}{1+sr_{c_2}C_2}\right) + \frac{sL_2+r_2}{R} + 1\right]}$$

$$= \frac{V_g\left[\frac{s^2L_1C_1}{1+sr_{c_1}C_1} + \frac{sr_1C_1}{1+sr_{c_1}C_1} + \frac{D(1-D)L_1 s}{R} + \frac{D(1-D)r_1}{R} + 1\right]}{\left[\frac{s^2L_1C_1}{1+sr_{c_1}C_1} + \frac{sr_1C_1}{1+sr_{c_1}C_1} + \frac{(1-D)^2L_1}{R}s + \underbrace{\frac{(1-D)^2r_1}{R} + 1}_{\approx 1}\right]\left[\frac{s^2L_2C_2}{1+sr_{c_2}C_2} + \frac{sr_2C_2}{1+sr_{c_2}C_2} + \frac{sL_2}{R} + \underbrace{\frac{r_2}{R} + 1}_{\approx 1}\right]}$$

$\times$ top and bottom by $(1+sr_{c_1}C_1)(1+sr_{c_2}C_2) \Rightarrow$

Also $r_1 \ll R \quad r_2 \ll R$

$$= \frac{V_g\left[s^2L_1C_1 + sr_1C_1 + \left[\frac{D(1-D)L_1 s}{R} + 1\right](1+sr_{c_1}C_1)\right](1+sr_{c_2}C_2)}{\left[s^2L_1C_1 + sr_1C_1 + \left[\frac{(1-D)^2L_1}{R}s + 1\right](1+sr_{c_1}C_1)\right]\left[s^2L_2C_2 + sr_2C_2 + \left[\frac{sL_2}{R} + 1\right](1+sr_{c_2}C_2)\right]}$$

$$= \frac{V_g \left\{ s^2\left[L_1C_1 + \frac{D(1-D)\, r_{C_1} L_1 C_1}{R}\right] + s\left[r_1 C_1 + r_{C_1} C_1 + \frac{D(1-D) L_1}{R}\right] + 1 \right\}\left(1 + s\, r_{C_2} C_2\right)}{\left(s^2\left[L_1C_1 + \frac{(1-D)^2 r_{C_1} L_1 C_1}{R}\right] + s\left[r_1C_1 + r_{C_1}C_1 + \frac{(1-D)^2 L_1}{R}\right] + 1\right)\left(s^2\left[L_2C_2 + \frac{r_{C_2} L_2 C_2}{R}\right] + s\left(r_2C_2 + r_{C_2}C_2 + \frac{L_2}{R}\right) + 1\right)}$$

$$\approx \frac{V_g \left[s^2 L_1C_1 + s\left[(r_1 + r_{C_1})C_1 + \frac{D(1-D) L_1}{R}\right] + 1\right]\left[1 + s\, r_{C_2} C_2\right]}{\left[s^2 L_1 C_1 + s\left[(r_1 + r_{C_1})C_1 + \frac{(1-D)^2 L_1}{R}\right] + 1\right]\left[s^2 L_2 C_2 + s\left[(r_2 + r_{C_2})C_2 + \frac{L_2}{R}\right] + 1\right]}$$

Main result: adds a zero to the transfer function due to the ESR of C2.

Resonant frequencies are unchanged but the Q factors do change, as there is greater damping in the circuit.

The output capacitor used: 47 micro Farad with 40 mOhm ESR :

A750EK476M1EAAE040

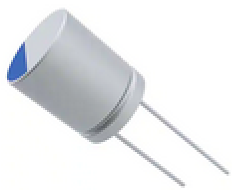


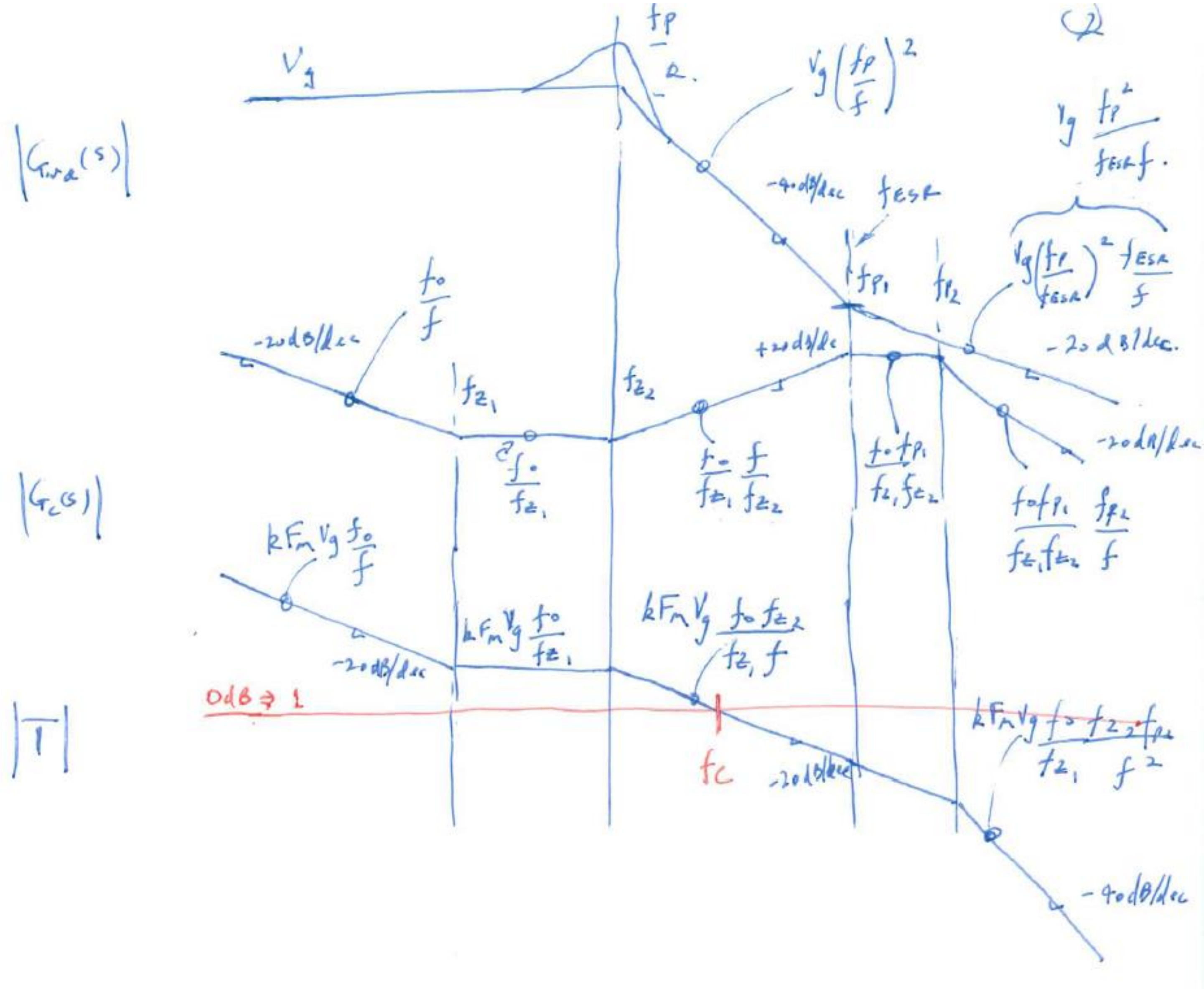
Image shown is a representation only. Exact specifications should be obtained from the product data sheet.

Digi-Key Part Number	399-13665-ND
Manufacturer	KEMET
Manufacturer Product Number	A750EK476M1EAAE040
Description	CAP ALUM POLY 47UF 20% 25V T/H
Manufacturer Standard Lead Time	20 Weeks
Detailed Description	47 µF 25 V Aluminum - Polymer Capacitors Radial, Can 40mOhm 2000 Hrs @ 105°C
Customer Reference	Customer Reference
Datasheet	Datasheet

Product Attributes

TYPE	DESCRIPTION	SELECT
Category	Capacitors Aluminum - Polymer Capacitors	
Mfr	KEMET	
Series	A750	
Package	Bulk	
Product Status	Active	
Type	Polymer	
Capacitance	47 µF	
Tolerance	±20%	
Voltage - Rated	25 V	
ESR (Equivalent Series Resistance)	40mOhm	

Compensator design:



Compensator transfer function, Gc(s), and resulting loop gain, T:

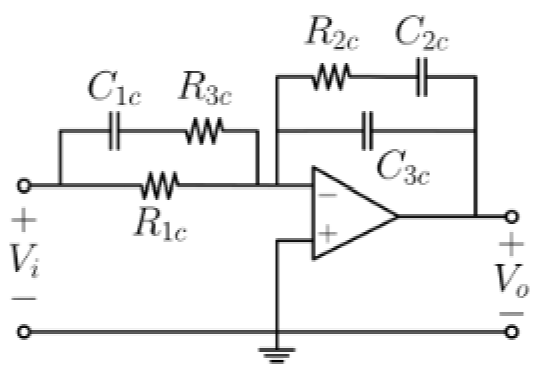
$$G_c(s) = \frac{\omega_o}{s} \frac{\left(1 + \frac{s}{\omega_{z_1}}\right)\left(1 + \frac{s}{\omega_{z_2}}\right)}{\left(1 + \frac{s}{\omega_{p_1}}\right)\left(1 + \frac{s}{\omega_{p_2}}\right)}$$

$$T = k\, F_m\, G_c(s)\, G_{vd}(s)$$

$$@\ f = f_c \Rightarrow \frac{k F_m V_g f_0 f_{z_2}}{f_{z_1} f_c} = 1$$

$$\Rightarrow f_0 = \frac{f_{z_1} f_c}{k F_m V_g f_{z_2}}$$

Compensator:



The transfer function $G_c(s)$ of this compensator is given by:

$$G_c(s) = -\frac{\omega_0}{s} \frac{\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}$$

where

$$\omega_0 = \frac{1}{R_{1c}(C_{2c}+C_{3c})}$$

$$\omega_{z1} = \frac{1}{R_{2c}C_{2c}}$$

$$\omega_{z2} = \frac{1}{C_{1c}(R_{1c}+R_{3c})}$$

$$\omega_{p1} = \frac{1}{R_{3c}C_{1c}}$$

$$\omega_{p2} = \frac{1}{R_{2c}\frac{C_{2c}C_{3c}}{C_{2c}+C_{3c}}}$$

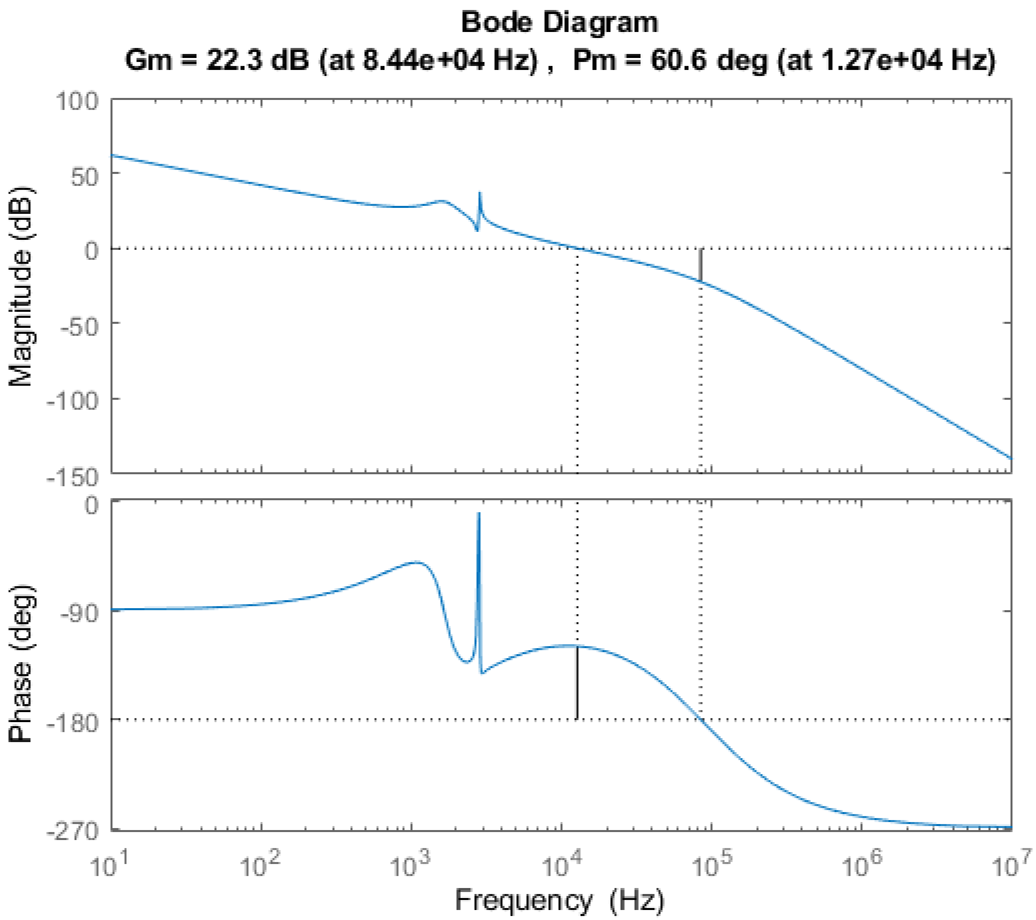
Matlab code:

```
% D4_RPT_class.m
% undertakes the D4 regulator design using a classical control approach

clear
close all
format short e

L1=33e-6          % D4 converter component values
L2=180e-6
C1=100e-6
C2=47e-6



Vg=10             % The parameters for converter
Vo=5
D=0.5  % = Vo/Vg
R=5               % nominal load resistance

k=1/5             % voltage divider gain
Vm=0.6            % pk-to-pk amplitude of triangular waveform in TL5001 IC
Fm=1/Vm           % modulator gain

esr=40e-3         % output capacitor ESR (equivalent series resistance)
fx=12.5e3         % desired crossover frequency 12,500 Hz

fesr=1/(2*pi*esr*C2)
Wesr=2*pi*fesr
Wn=1/sqrt(L2*C2) % (dominant) corner frequency
fp=Wn/(2*pi)
fz2=fp
alpha=1           % alpha is between 0.3-1, choose alpha=1
fz1=alpha*fp
fp1=fesr
fp2=90e3          % Choose the fp2 at 90KHz
f0=(fz1*fx)/(k*Fm*Vg*fz2)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

W0=2*pi*f0        % the parameters for the compensator
Wz1=2*pi*fz1
Wz2=2*pi*fz2
Wp1=2*pi*fp1
Wp2=2*pi*fp2

%Plot the phase margin and crossover frequence
s=tf('s');
% compensator TF
Gc=(W0/s)*(((1+s/Wz1)*(1+s/Wz2))/((1+s/Wp1)*(1+s/Wp2)))

% control to output TF of converter
Gvd_num = Vg*(C1*L1*s^2 + D*(1-D)*L1/R*s + 1);
Gvd_den = C1*C2*L1*L2*s^4  +  C1*L1*L2/R*s^3 +  ...
          ((1-D)^2*C2*L1+C2*L2+C1*L1)*s^2 + ((1-D)^2*L1+L2)/R*s + 1;
Gvd = Gvd_num/Gvd_den;


figure(1)
Gvd_zpk = zpk(Gvd)
Gvd_zpk_minreal = minreal(Gvd_zpk, 0.06)  % cancellation
```

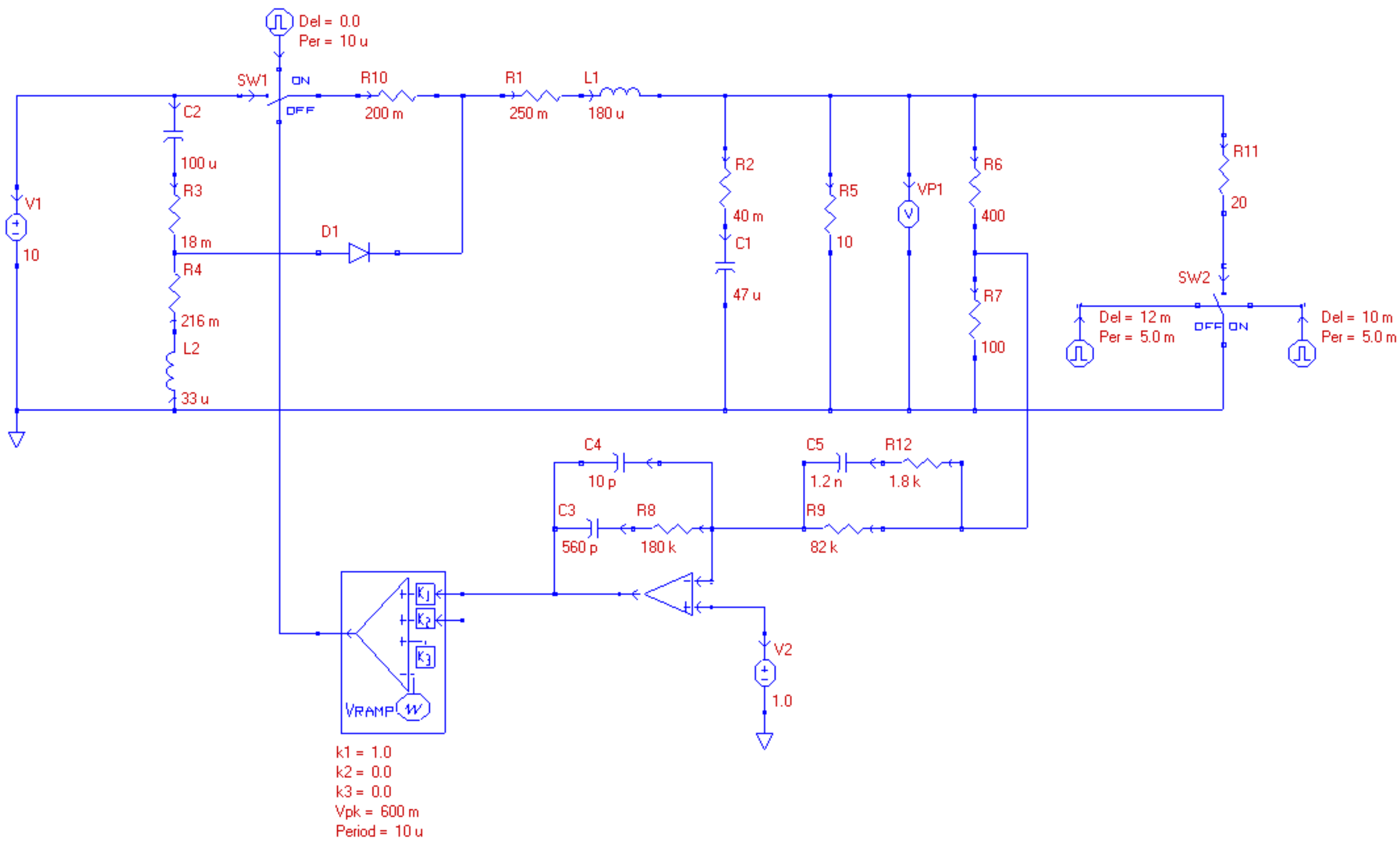
```
bode(Gvd,Gvd_zpk_minreal)
h = gcr;                    % Displayed the crossover frequency in Hz
h.AxesGrid.Xunits = 'Hz';


figure(2)
Loopgain=k*Gc*Fm*Gvd
margin(Loopgain)
h = gcr;                    % Displayed the crossover frequency in Hz
h.AxesGrid.Xunits = 'Hz';


Gc_zpk = zpk(Gc)            % compensator TF


Gvd_den_approx = (L1*C1*s^2+(1-D)^2*L1/R*s+1) * (L2*C2*s^2+L2/R*s+1);
Gvd_approx = Gvd_num/Gvd_den_approx;
figure(3)
bode(Gvd,Gvd_approx)
h = gcr;                    % Displayed the crossover frequency in Hz
h.AxesGrid.Xunits = 'Hz';

%%%%%%%%%%%%%%%%%%%%%%%%%%

% % with added damping (i.e. r and C3) to L1, C1
%
% C1=10e-6
% L1=33e-6
% C2=10e-6
% L2=180e-6
% r = 0.5
% C3 = 22e-6
%
% A1 = [0       0       1/L1        0           0
%       0       0       0          -1/L2        0
%      -1/C1    0      -1/(r*C1)    0           1/(r*C1)
%       0       1/C2    0          -1/(R*C2)    0
%       0       0       1/(r*C3)    0          -1/(r*C3)];
%
% B1 = [-1/L1;   1/L2;  0;  0;  0];
%
% A2 = [0       0       1/L1        0           0
%       0       0      -1/L2       -1/L2        0
%      -1/C1    1/C1   -1/(r*C1)    0           1/(r*C1)
%       0       1/C2    0          -1/(R*C2)    0
%       0       0       1/(r*C3)    0          -1/(r*C3)];
%
% B2 = [-1/L1;   1/L2;  0;  0;  0];
%
%
% A = D*A1+(1-D)*A2;
% B = D*B1+(1-D)*B2;
% C = [0, 0, 0, 1, 0];
%
% X = -A\B*Vg;
% Bd = (A1-A2)*X+(B1-B2)*Vg;
%
% sys_bd = ss(A,Bd,C,0);
% figure(4)
% bode(sys_bd, Gvd, {2*pi*100, 2*pi*1e5})
% h = gcr;
```

```
% h.AxesGrid.Xunits = 'Hz';
%
% sys_zpk = zpk(sys_bd)
% sys_zpk = minreal(zpk(sys_bd), 0.02)
% sys_zpk = minreal(zpk(sys_bd), 0.05)
%
% figure(5)
% margin(k*Gc*Fm*sys_bd)
% h = gcr;
% h.AxesGrid.Xunits = 'Hz';

% % sys_bd_zpk = zpk(sys_bd)
% % sys_bd_zpk_minreal = minreal(sys_bd_zpk, 0.05)
% % figure(4)
% % bode(sys_bd_zpk_minreal)
% % h = gcr;
% % h.AxesGrid.Xunits = 'Hz';

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% calculate the compensator component values
% Gc=(W0/s)*(((1+s/Wz1)*(1+s/Wz2))/((1+s/Wp1)*(1+s/Wp2)))

C3 = 10e-12
R2 = 1/(Wp2*C3)
C2 = 1/(Wz1*R2)
R1 = 1/(W0*(C2+C3))
C1 = 1/R1 * (1/Wz2 - 1/Wp1)
R3 = 1/(Wp1*C1)
disp('*******')

% % results:
% C3 = 1.0000e-11   % 10 pF
% R2 = 1.7684e+05   % 180 kOhm
% C2 = 5.2012e-10   % 560 pF
% R1 = 8.0059e+04   % 82 kOhm
% C1 = 1.1254e-09   % 1.2 nF
% R3 = 1.6705e+03   % 1.8 kOhm

%%%%%%%%%%%%%%%%%%%%%%%

%  Or, swapping Wz1 and Wz2 (if they are different, depends on alpha above)
% C3a = 10e-12
% R2a = 1/(Wp2*C3a)
% C2a = 1/(Wz2*R2a)
% R1a = 1/(W0*(C2a+C3a))
% C1a = 1/R1a * (1/Wz1 - 1/Wp1)
% R3a = 1/(Wp1*C1a)
```

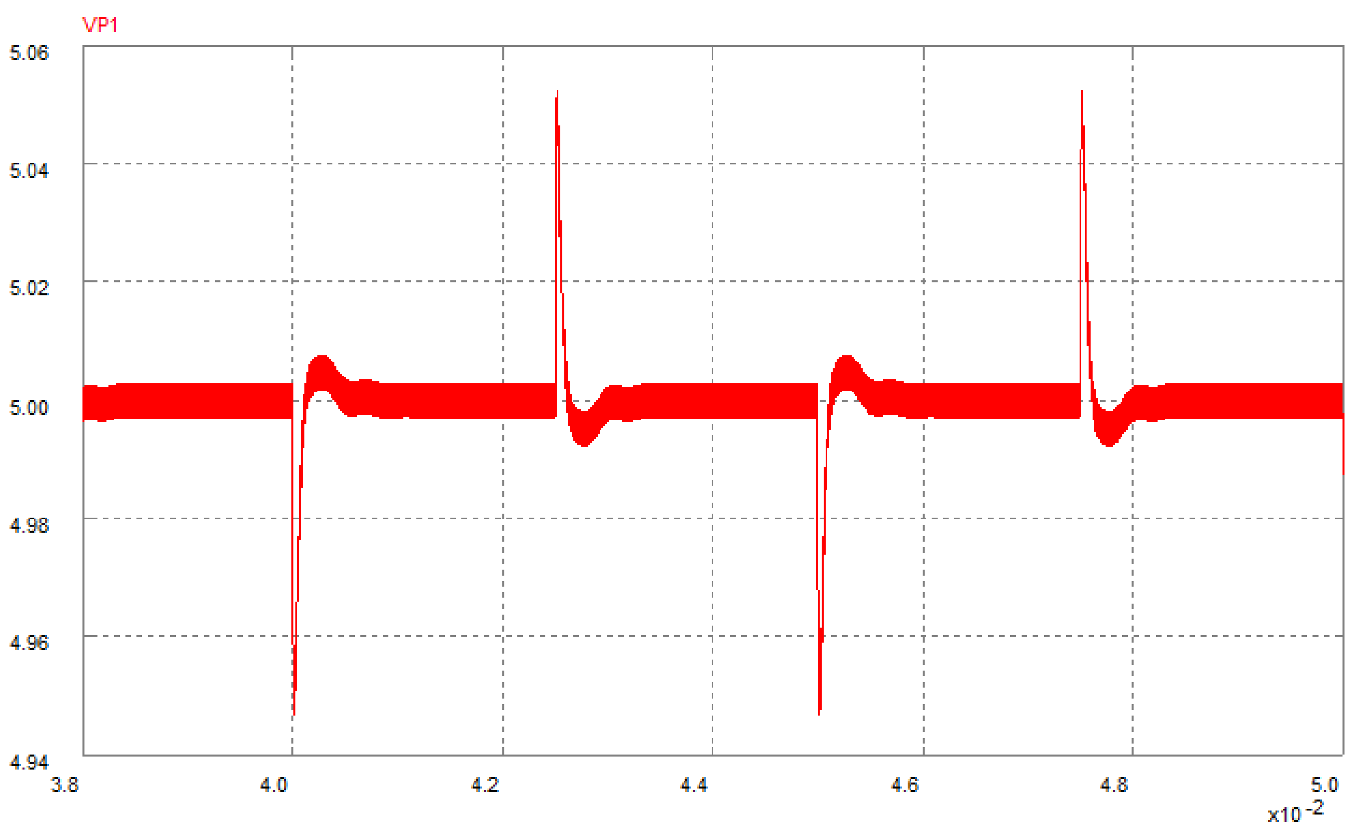
Loop gain Bode plot:



PECS implementation:



Step load response:



Step input voltage response: 10 V -> 11 V -> 10 V